



- Single-layer steel grid shells -  
Behaviour study and grid pattern  
optimization

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# Single-layer steel grid shells

Free-form structures

Roof structures, triangular network

Shell+beam-like behaviour

Shell buckling, snap-through



## Can we improve the structural behaviour by changing the geometry?

Geometry: beam length, angles, mesh density, topology



Improvement in structural behaviour: maximizing load-bearing capacity

*Mesh, grid:* network of beam centrelines

Load bearing capacity:

Nonlinear, numerical analysis

Moving the nodes along a predefined surface:

Fix - surface, topology,  
boundary nodes

Variables - inner nodes



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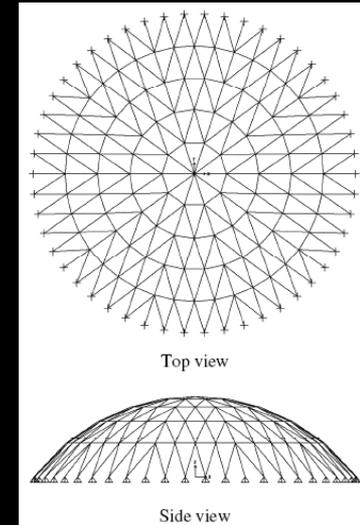
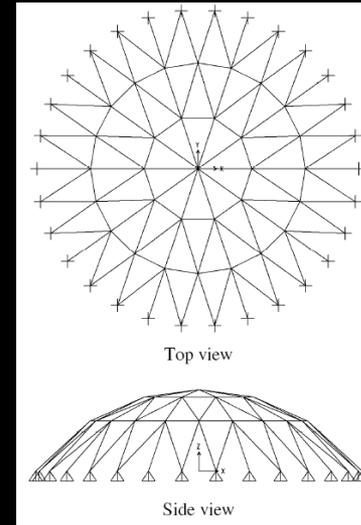
Load bearing capacity:

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Variables - inner nodes



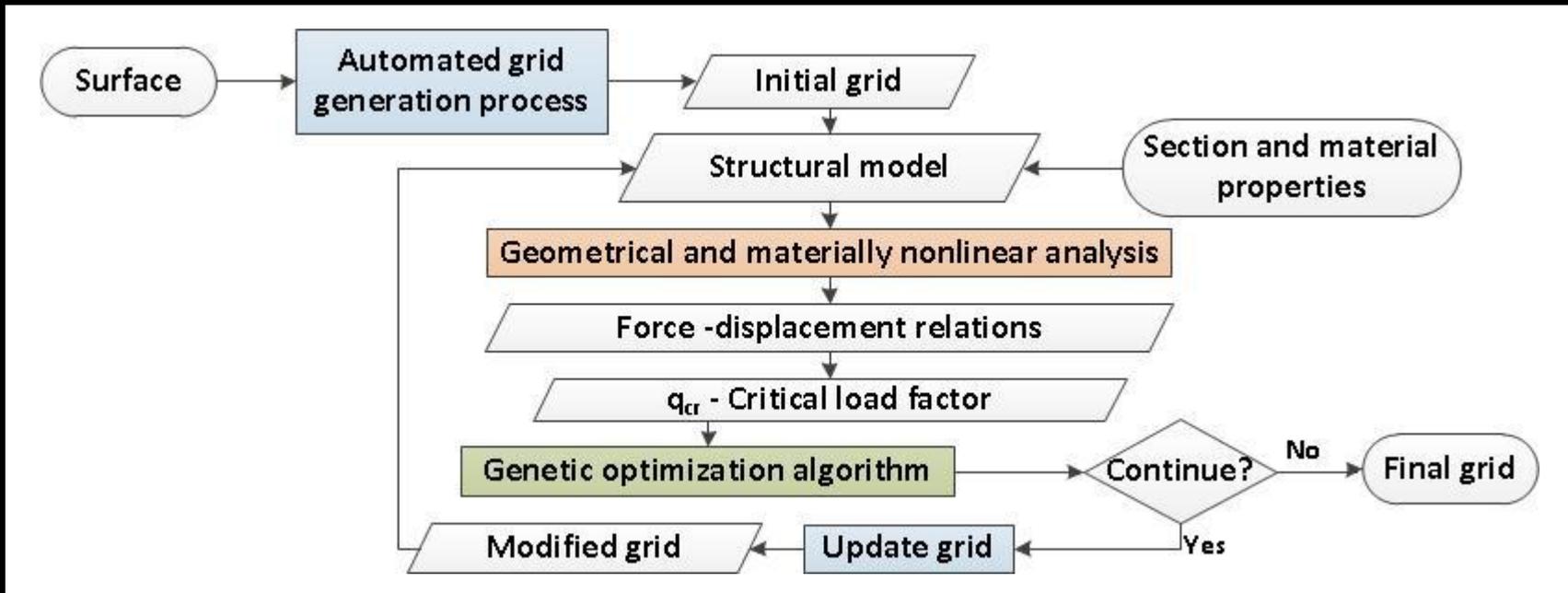
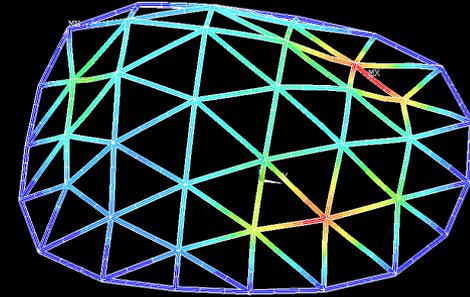
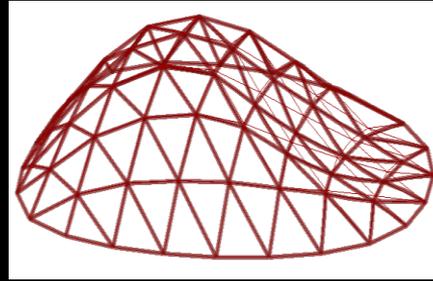
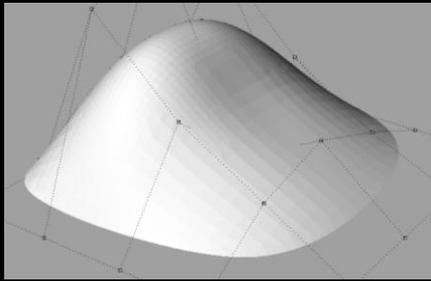
[Kaveh, 2011]

Topology and size optimization in literature: simple mesh types, simple surfaces



Mesh generating algorithm

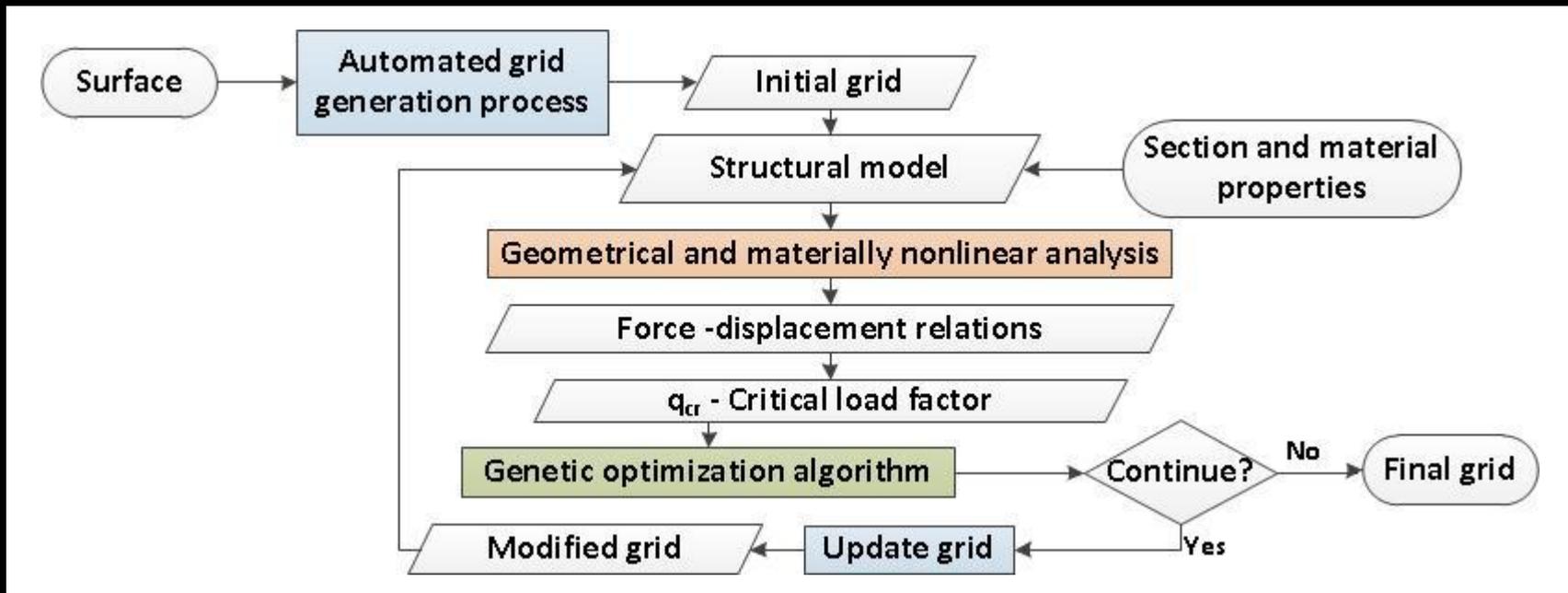
## Process and steps of grid pattern optimization



## Process and steps of grid pattern optimization

### Contents of presentation:

- Analysis - Structural model
- Analysis - Solver
- $q_{cr}$  : load bearing capacity
- Load, fitness function
- Automated grid generation process
- Results - domes, free-form surfaces



## Finite element model

Beam finite elements

Rigid nodes, fixed supports

Perfectly elasto-plastic material model

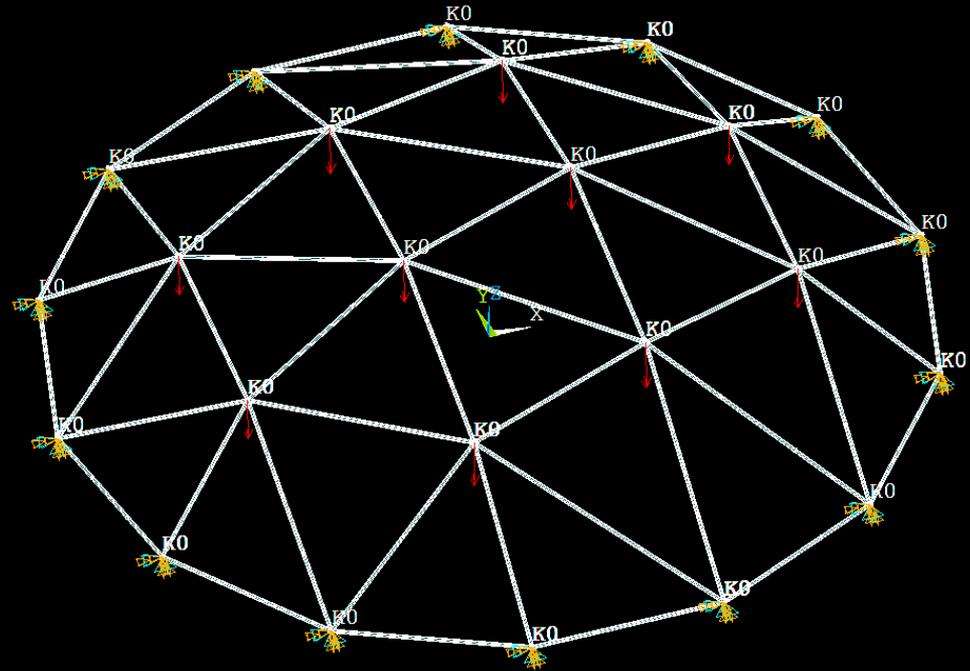
Vertical nodal loads

Section: pipe: CHS 146\*5 ( $r=73\text{mm}$ ,  $t=5\text{mm}$ )

Steel grade: S235

Plasticity is not a typical failure mode

Beam length,  $\lambda_{\text{rel}} = 1 \div 1,8$



## Arc-length method

Geometrically nonlinear analysis,  
no imperfections

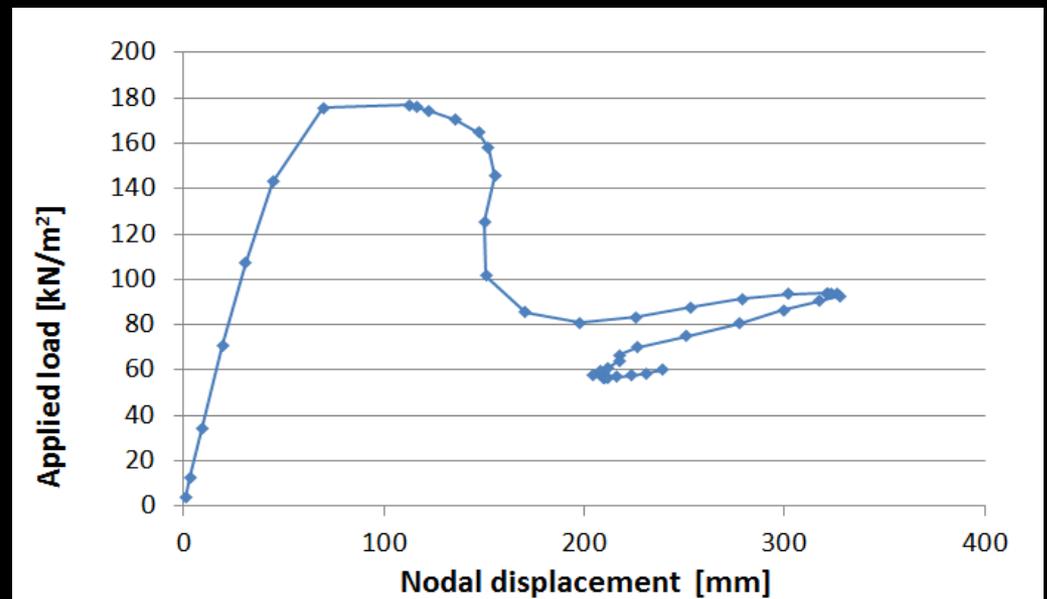
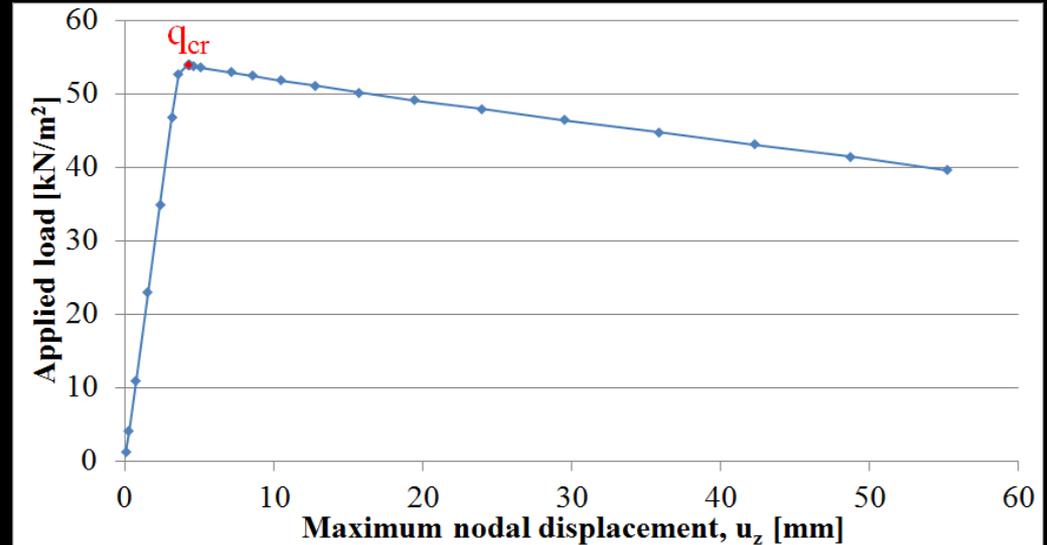
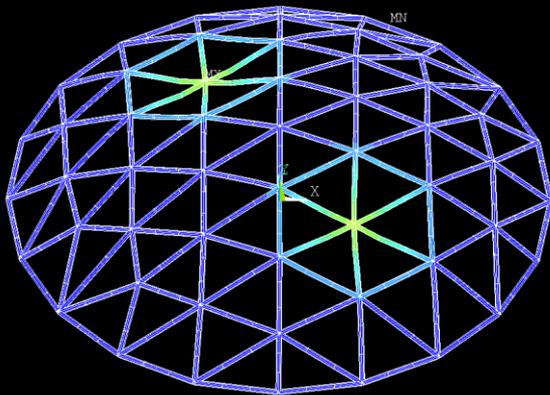
Unstable behaviour

Load displacement curves →  $q_{cr}$

ANSYS

Radius of arc-length:

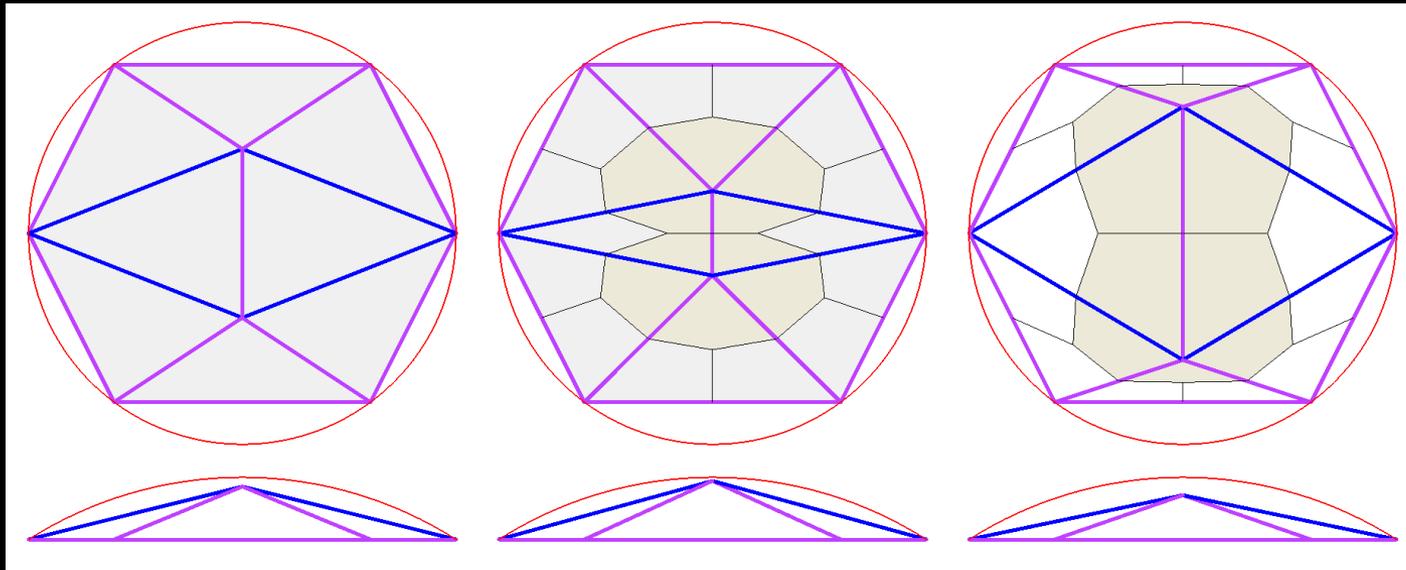
- exact maximal load
- post-critical behaviour



## Vertical nodal loads

Uniform nodal loads

Uniform distributed load – transferred to nodes based on triangular areas



Fitness function:

$$F_{cr} \text{ [kN]}$$

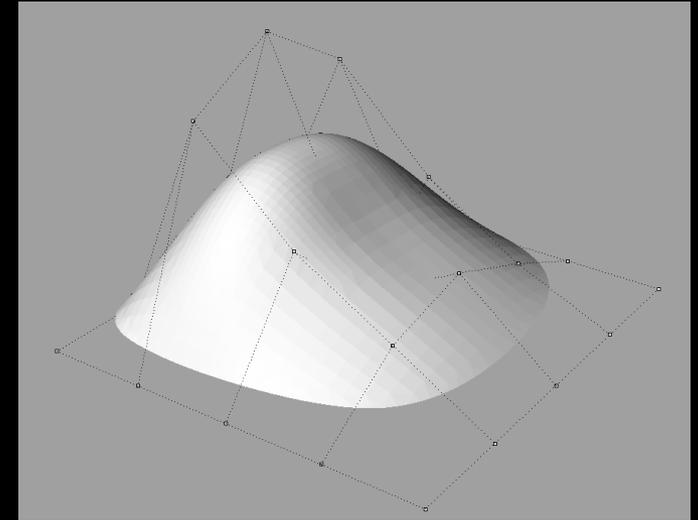
$$q_{cr} \text{ [kN/m}^2\text{]}$$

$$q_{cr} * A_{inner} / A \text{ [kN/m}^2\text{]}$$

## Automated mesh generation

### Goal

- mesh – beam centrelines
- applicable for free form surfaces (NURBS)
- equidistant supports

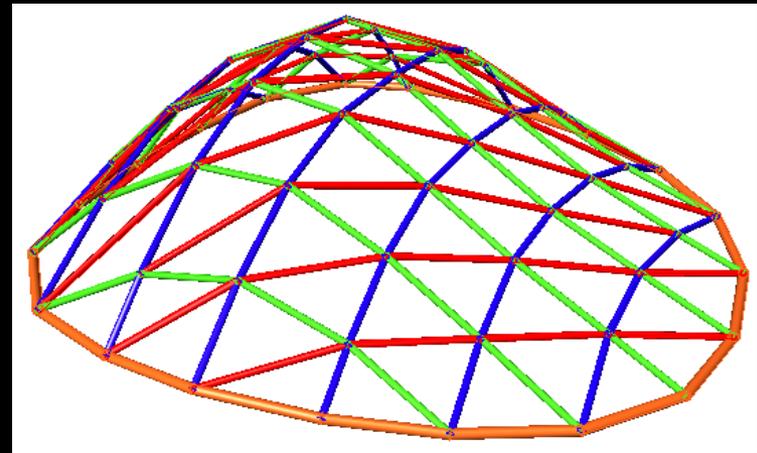
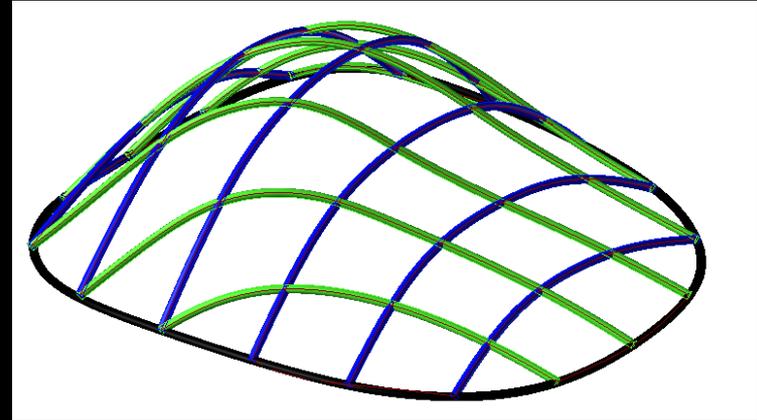


## Automated mesh generation

### Goal

- mesh – beam centrelines
- applicable for free form surfaces (NURBS)
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Method: slicing the surface with 2 sets of bent planes



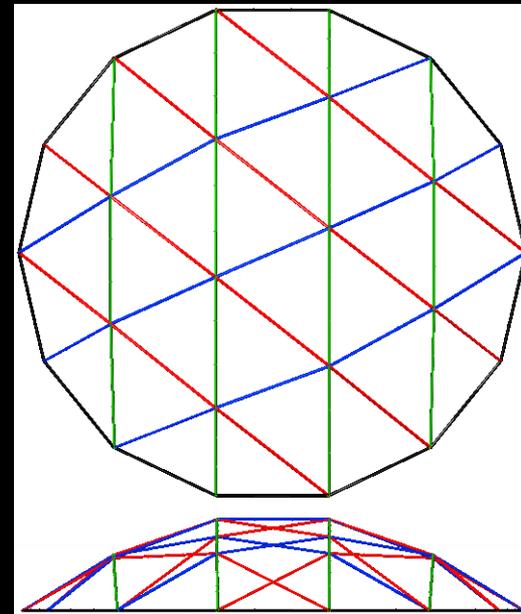
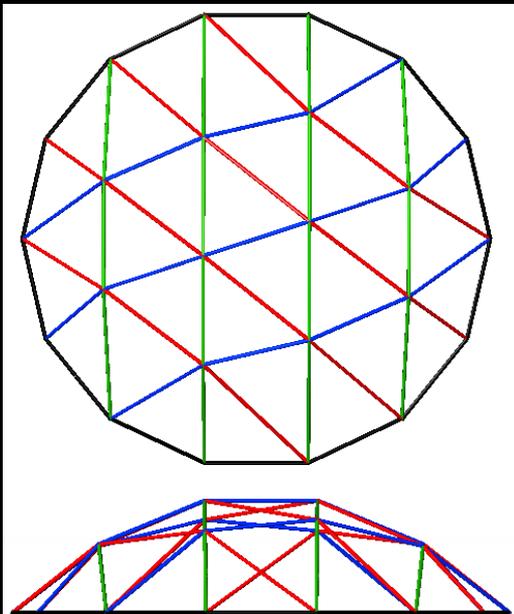
## Automated mesh generation

### Goal

- mesh – beam centrelines
- applicable for free form surfaces (NURBS)
- equidistant supports

Method: slicing the surface with 2 sets of bent planes

Relaxation: refining the initial grid



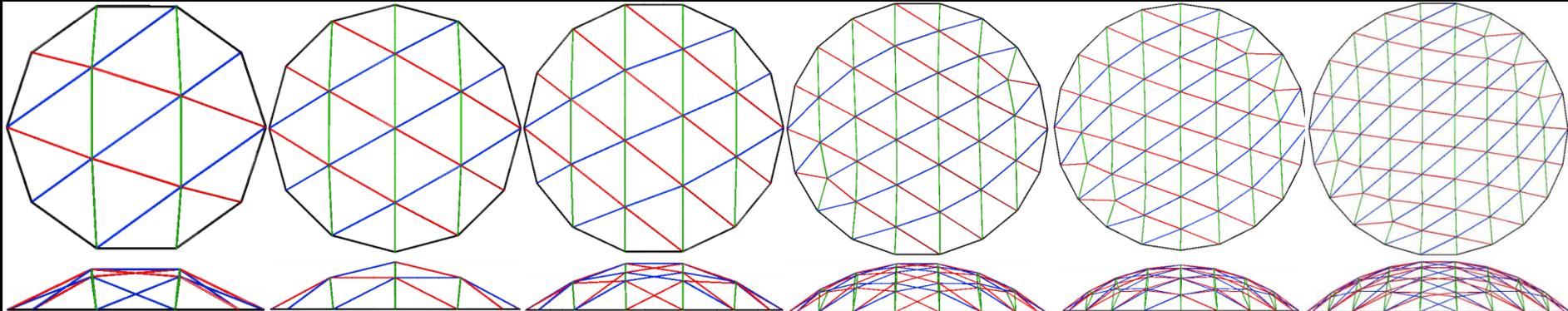
## Automated mesh generation - generated meshes

Basic parameter: division number:  $n$



mesh density, coarseness

Dome:



$n = 3$

$n = 4$

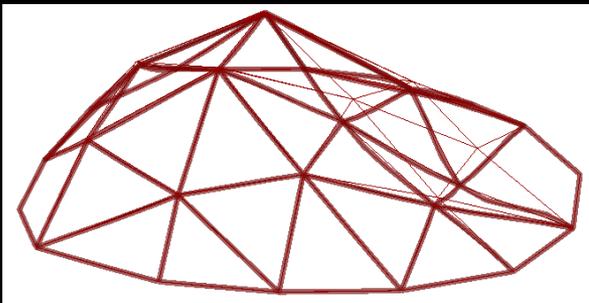
$n = 5$

$n = 7$

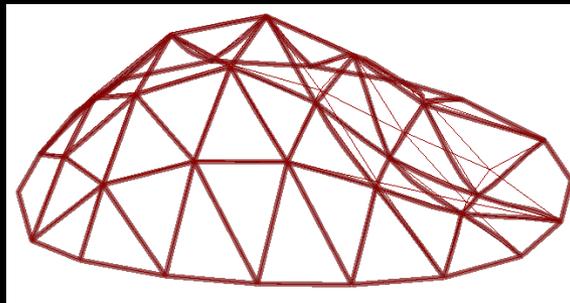
$n = 8$

$n = 9$

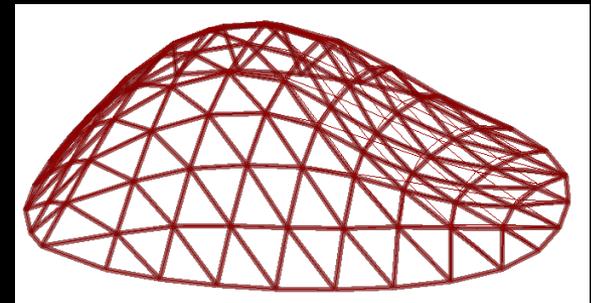
Free-form:



$n = 5$



$n = 7$



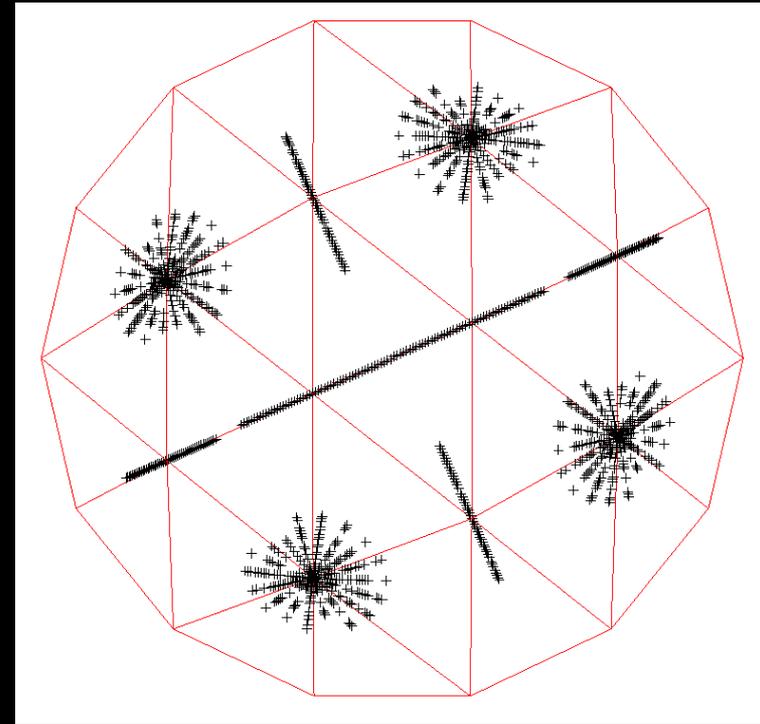
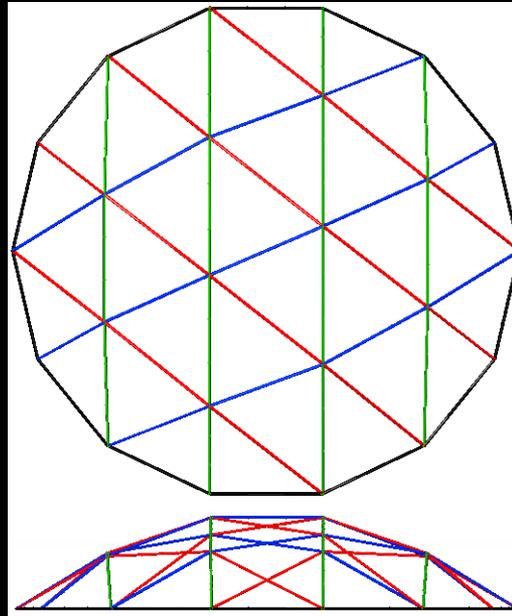
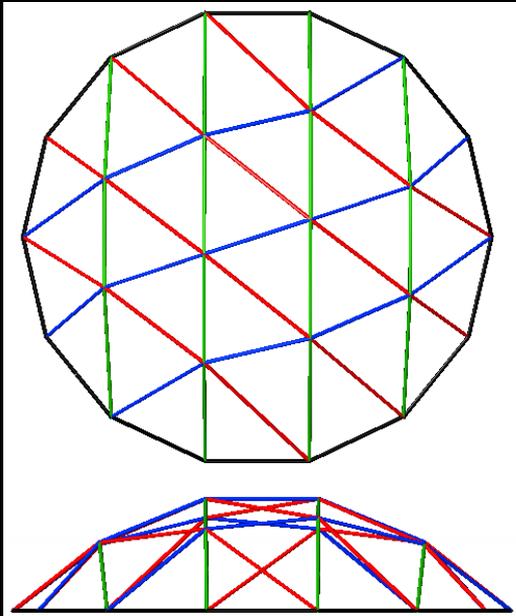
$n = 10$

Coarse mesh

Generated mesh – relaxed mesh

No difference in qcr

Double symmetric layout – nodes are constrained



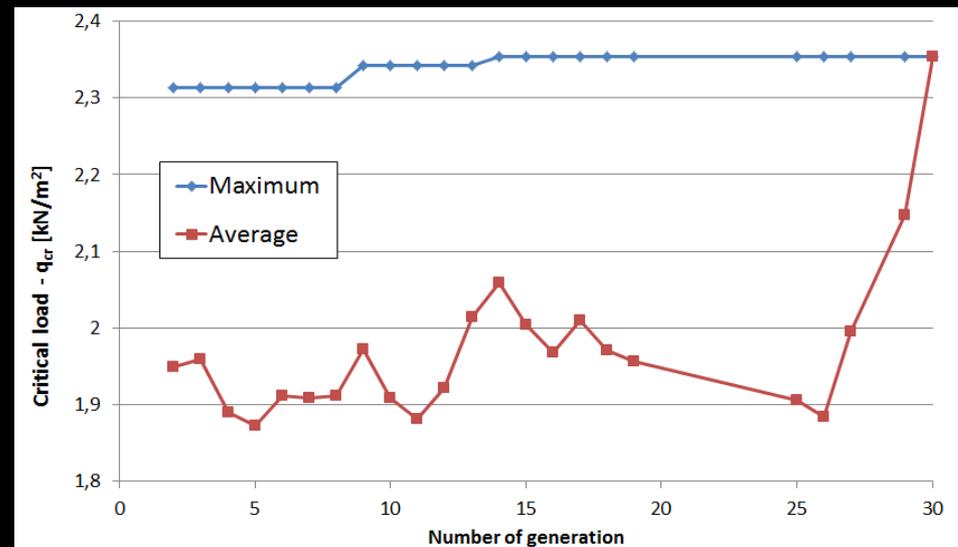
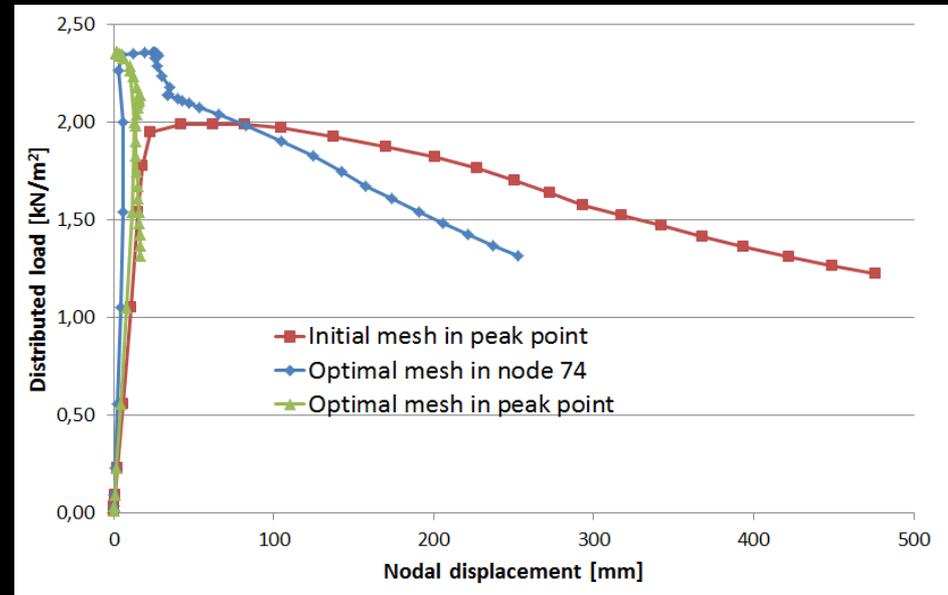
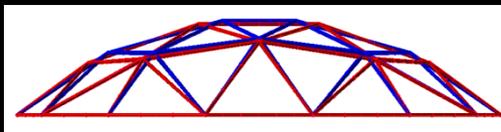
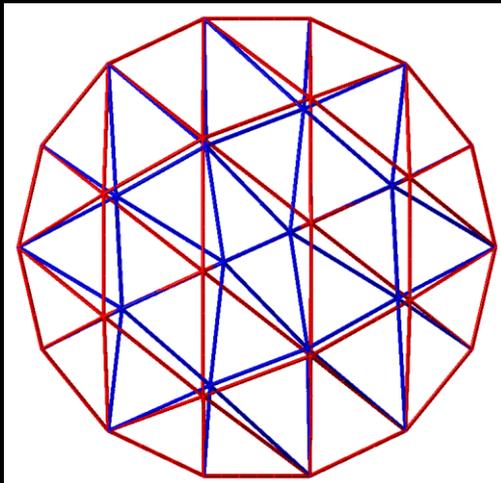
# Results – Coarse mesh – Dome, n= 10

## Coarse mesh

Improvement: 18 %

Initial mesh

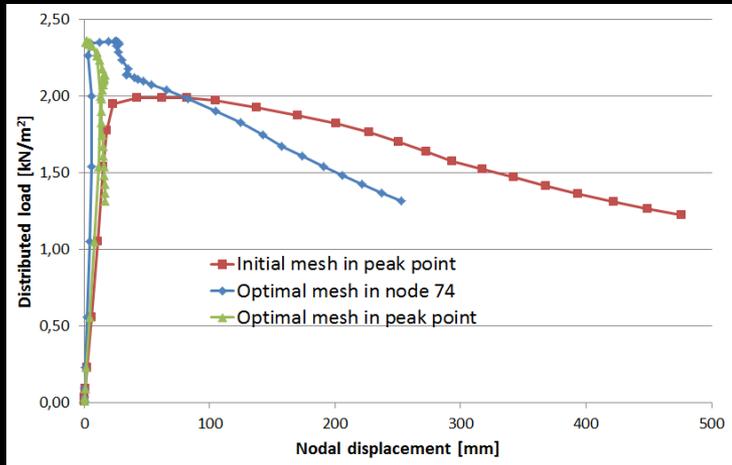
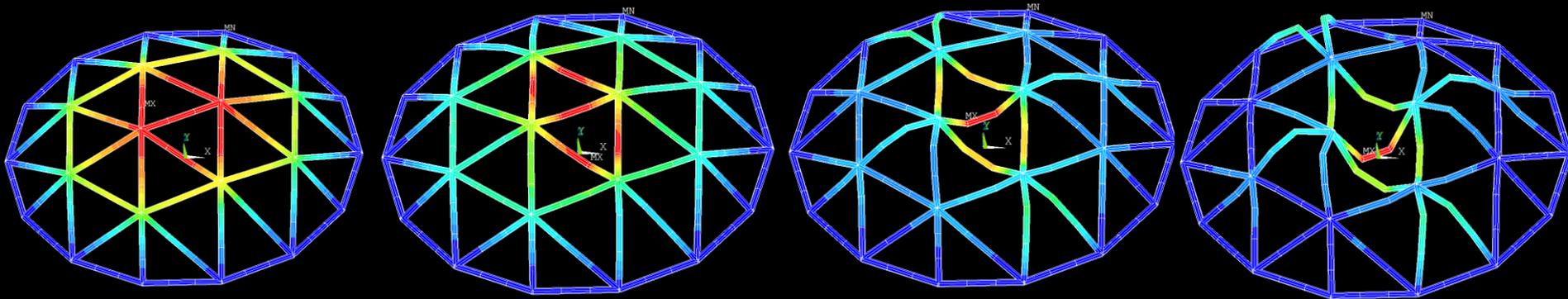
Optimal mesh



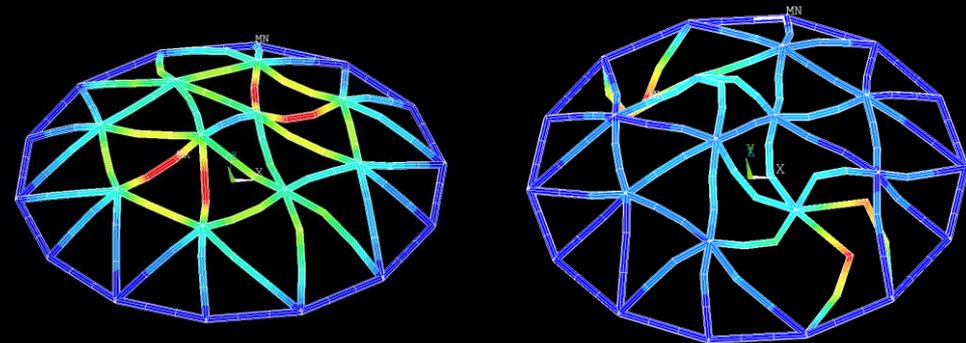
Coarse mesh

Different failure modes

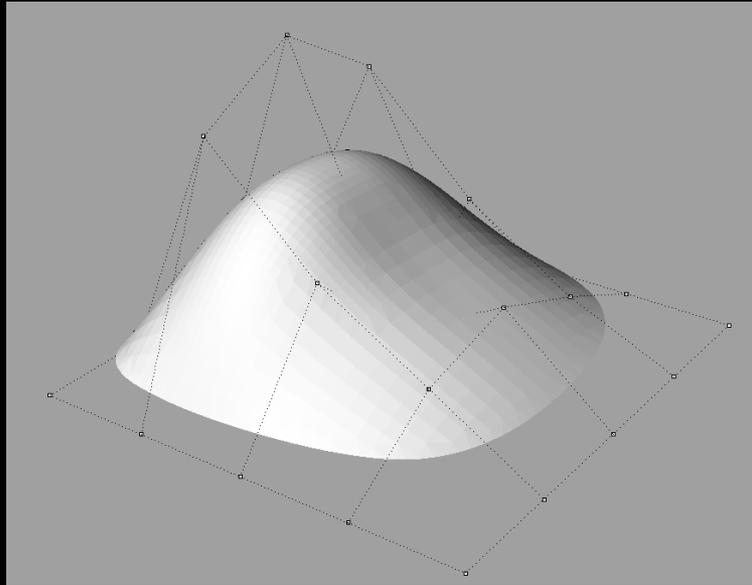
Initial



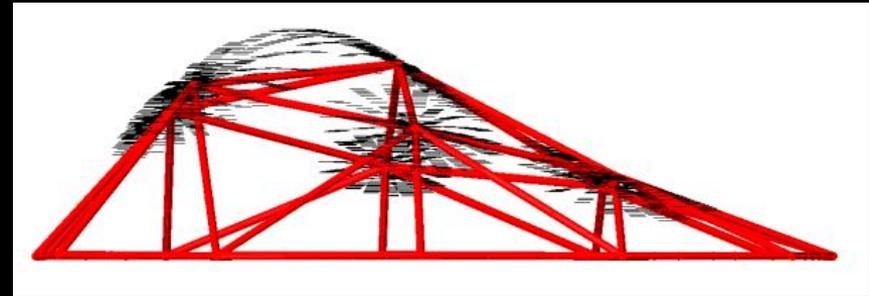
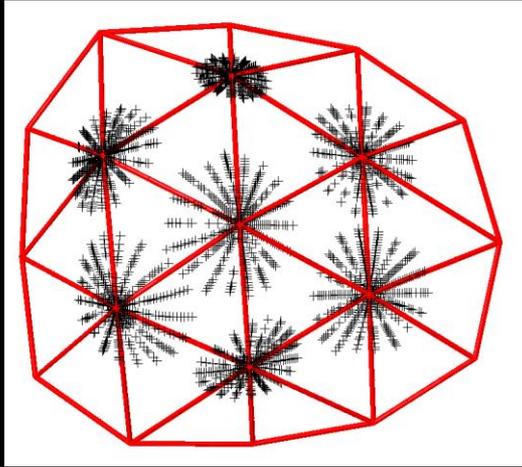
Optimal



Coarse mesh

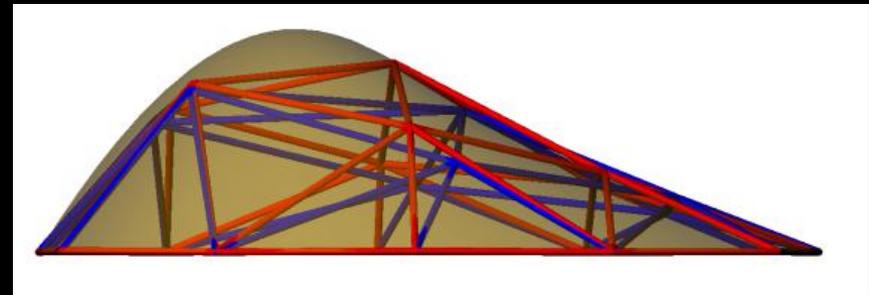
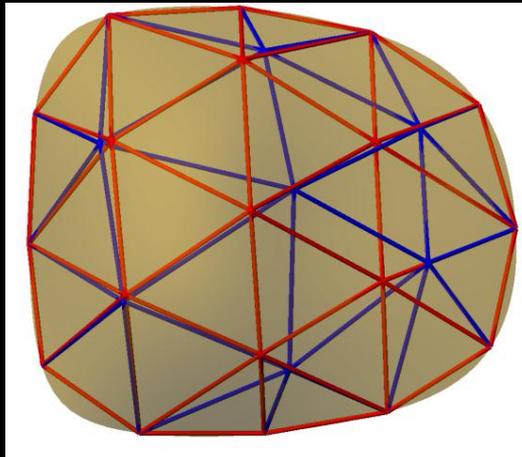


Coarse mesh

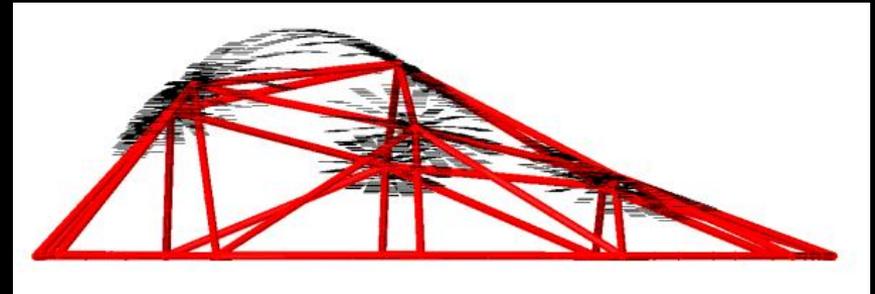
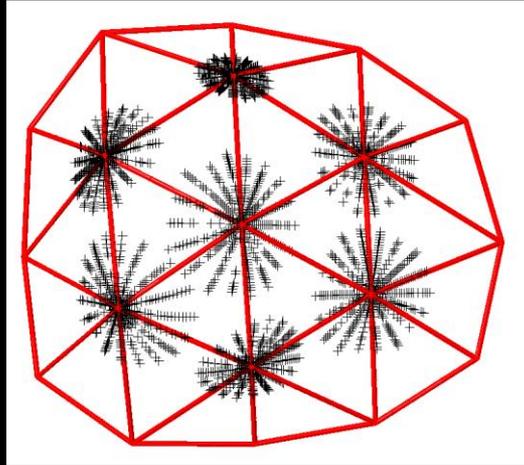


Initial mesh  
Optimal mesh

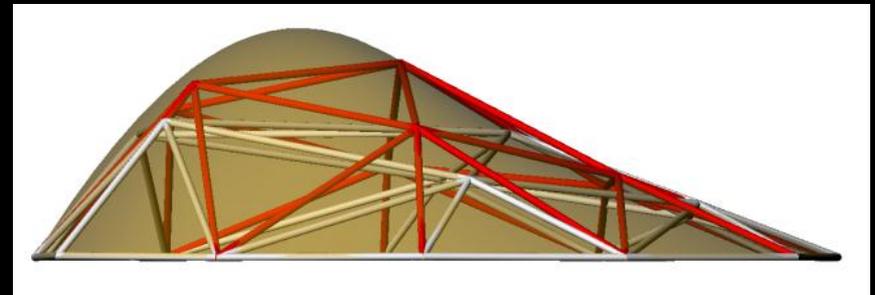
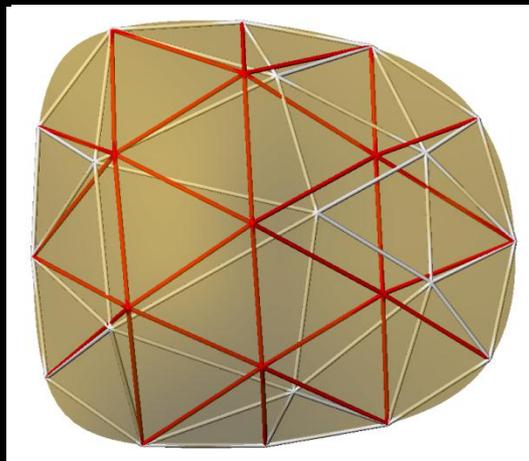
28 %



Coarse mesh



Initial mesh  
Optimal mesh

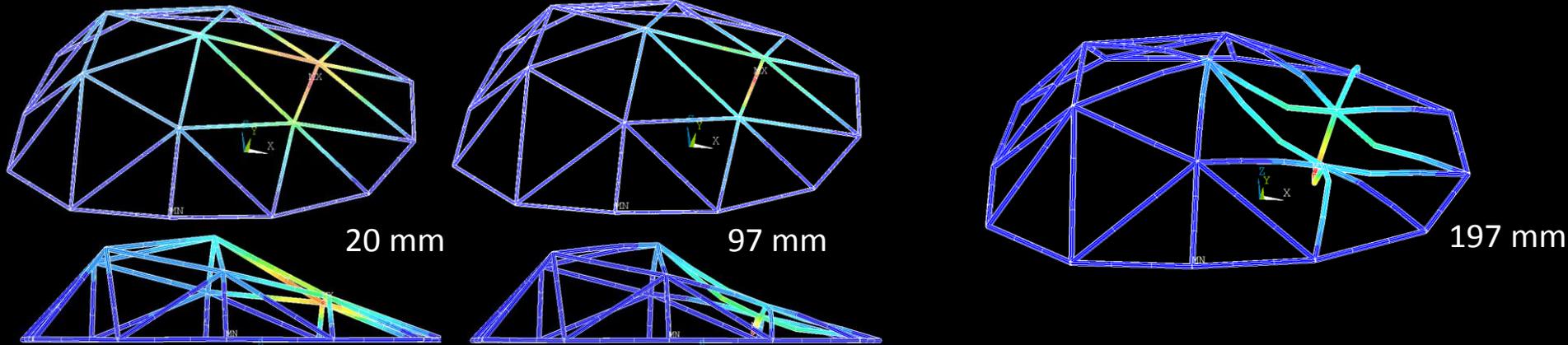


35 %

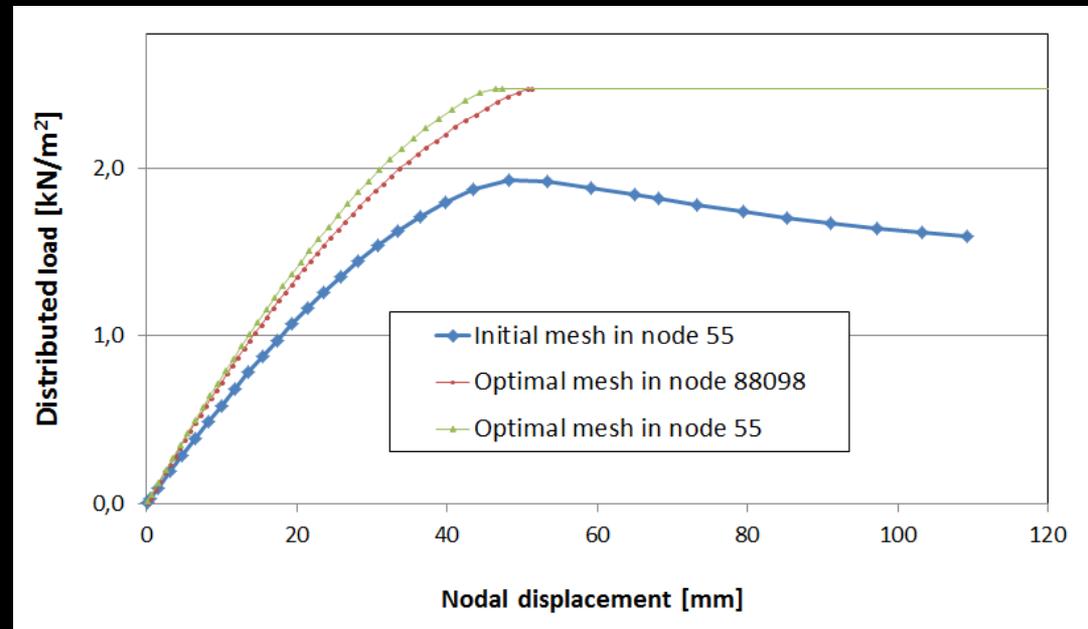
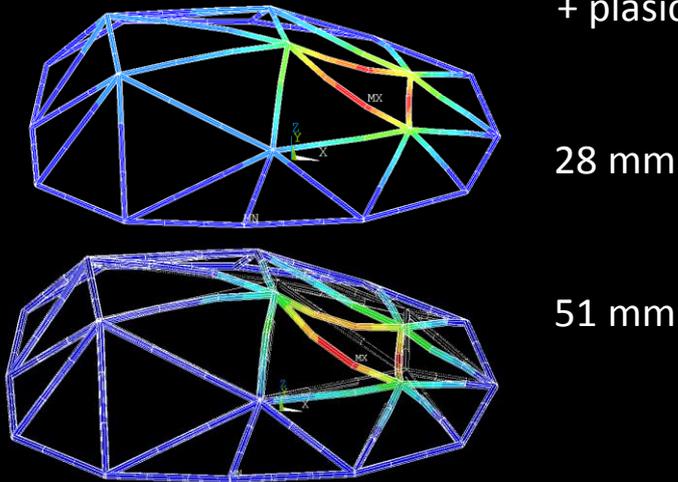
# Results – Coarse mesh – Free-form #1, n=7

## Different failure modes

Initial mesh – element buckling

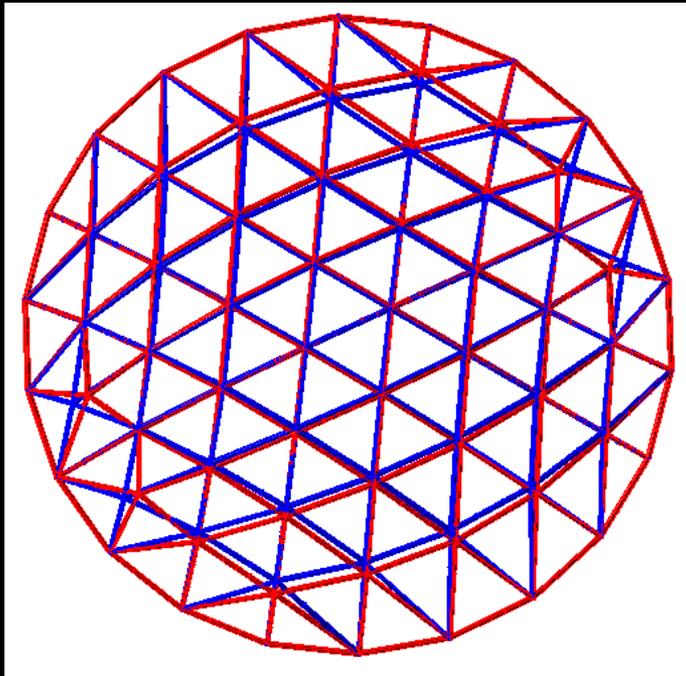


Optimal mesh – snap through of 3 nodes + plasticity

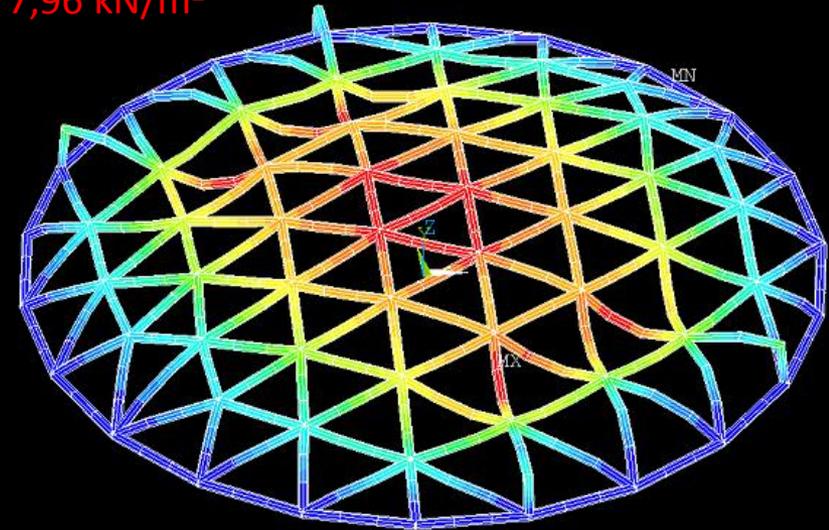


Failure mode: plasticity

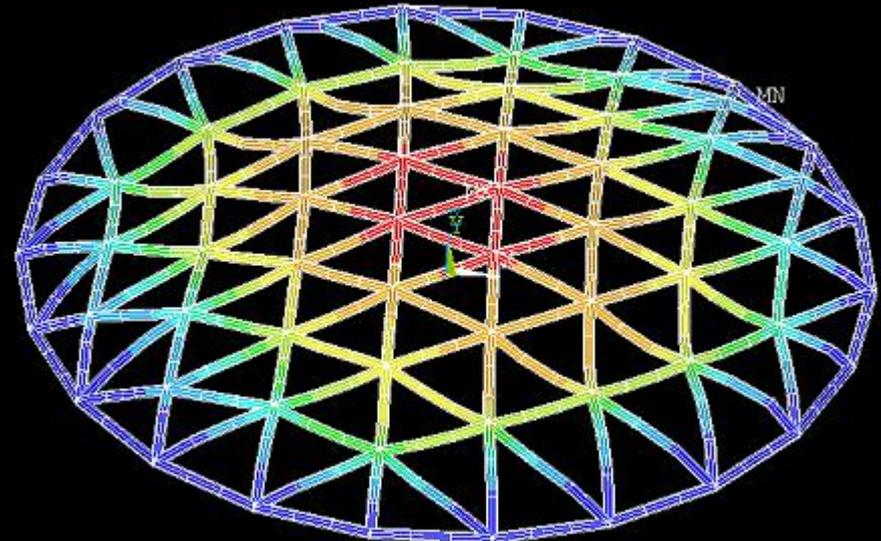
Initial - 7,96 kN/m<sup>2</sup>



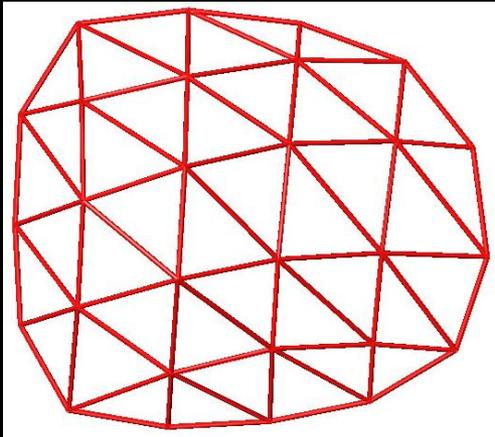
4 %



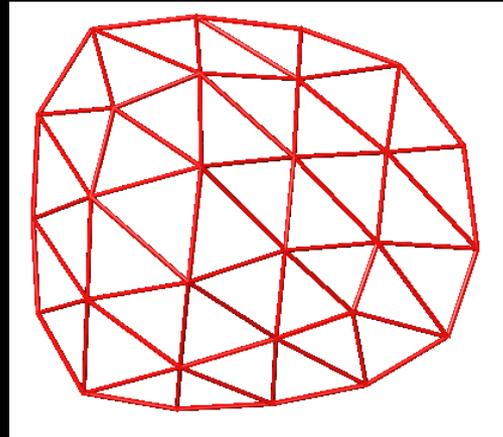
Relaxed = Optimal - 8,21 kN/m<sup>2</sup>



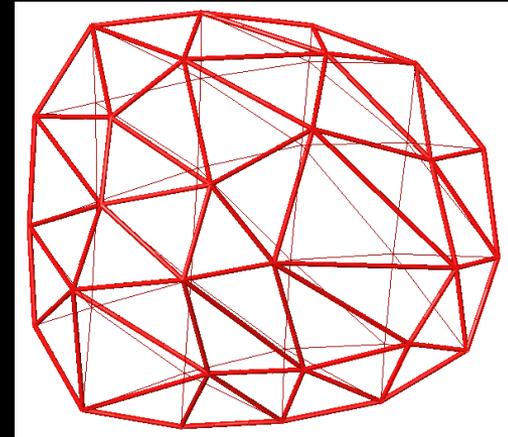
# Results – Free-form #1, n=14



Initial (generated) 2,06 kN/m<sup>2</sup>



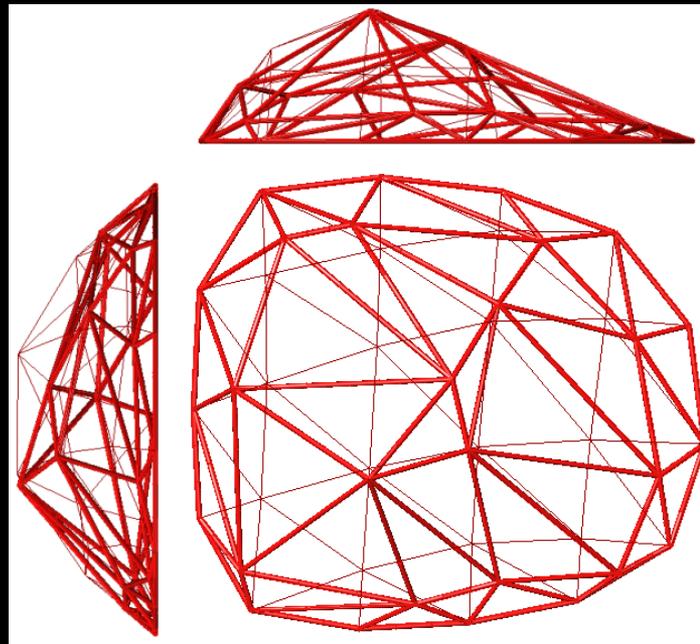
Relaxed 2,14 kN/m<sup>2</sup>

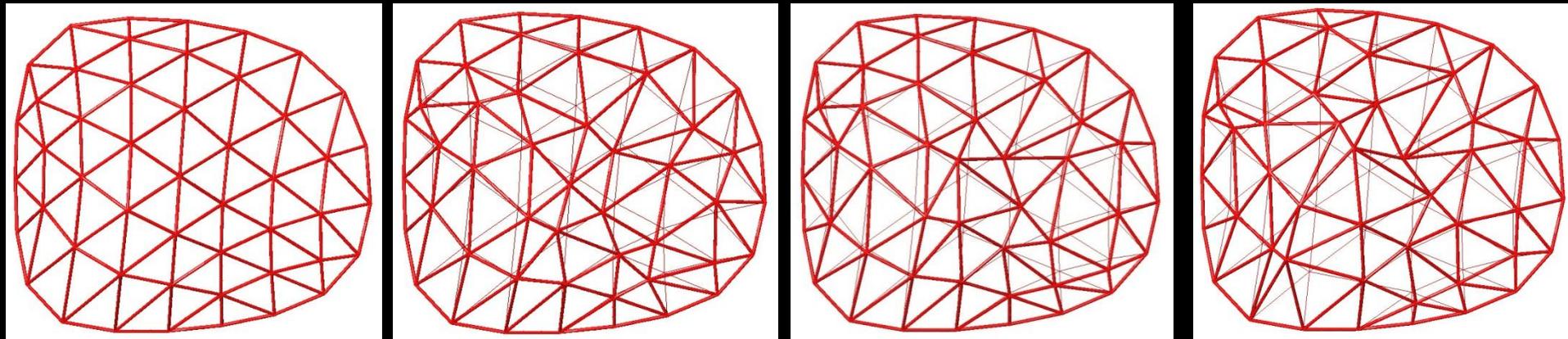


Modified 3,20 kN/m<sup>2</sup>

Optimal 4,11 kN/m<sup>2</sup>

100 %





Initial mesh (unrelaxed)  
3,56 kN/m<sup>2</sup>

→ 3,96 kN/m<sup>2</sup>

→ 4,10 kN/m<sup>2</sup>

→ Optimal  
4,23 kN/m<sup>2</sup>

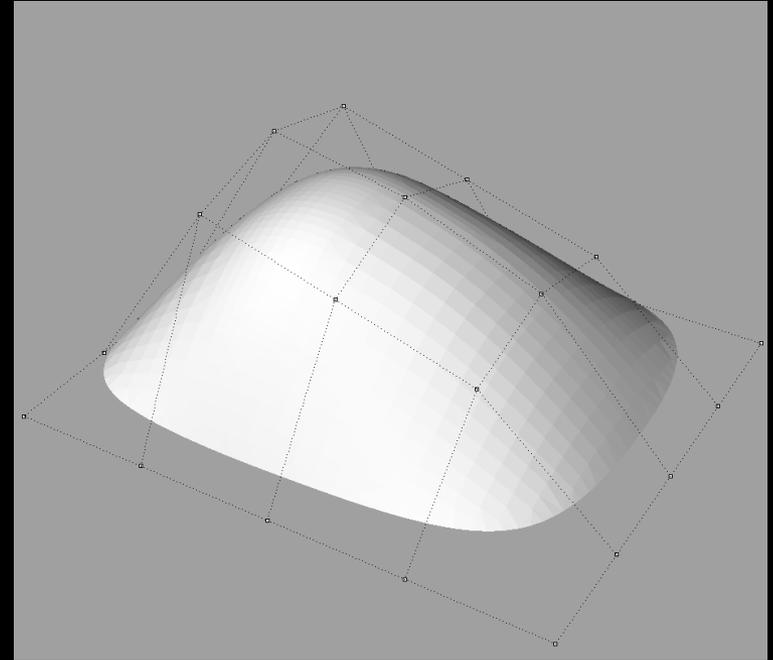
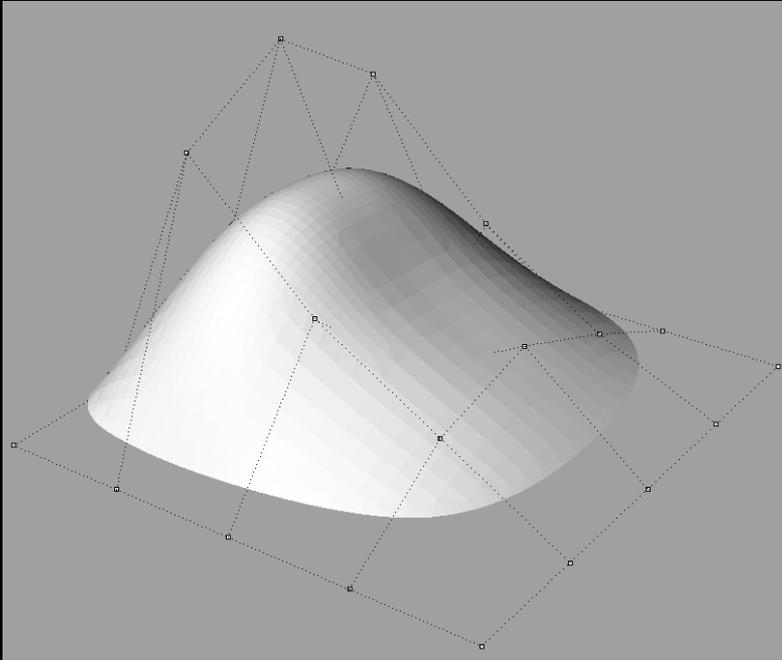
19 %

Failure mode:

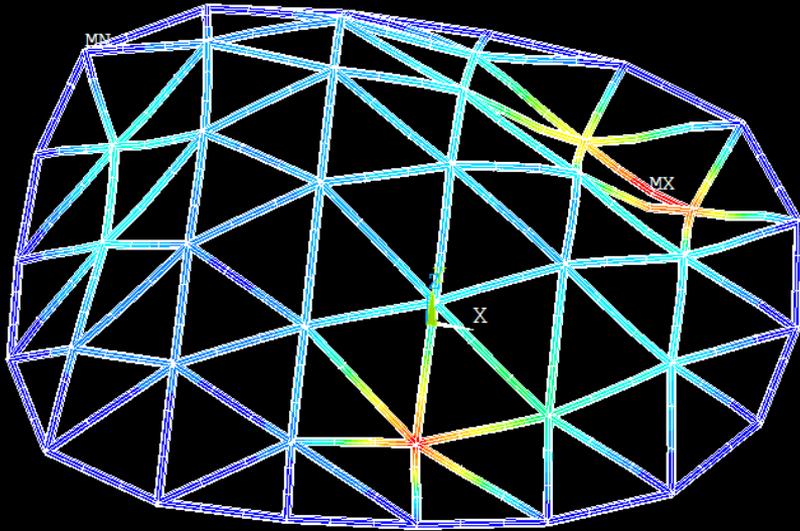
Shell buckling (many nodes involved) due to very shallow surface  
Nodes can not move away from here

Different initial surface suggested

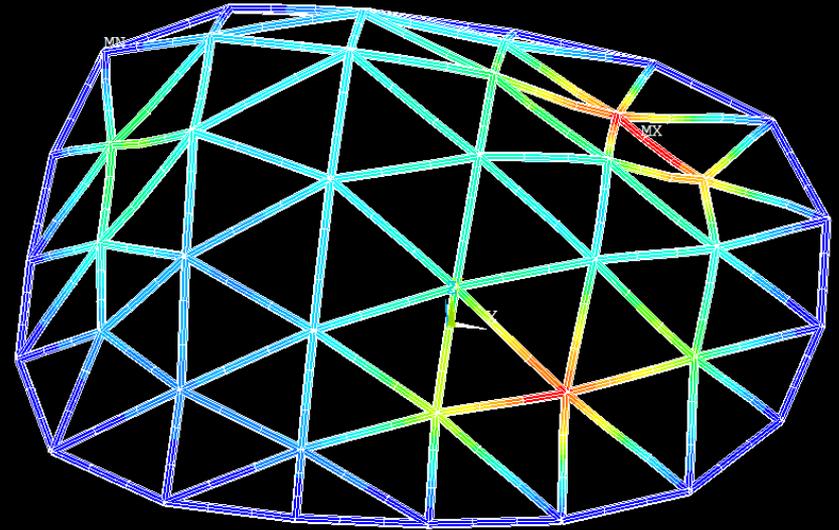
## Modifying the surface NURBS control points



## Plastic failure



Initial mesh - 5,45 kN/m<sup>2</sup>



Relaxed mesh - 7,06 kN/m<sup>2</sup>

Optimal mesh - 7,16 kN/m<sup>2</sup>

31 %

Developed a method for grid pattern optimization

The significant effect of member grid pattern on load bearing capacity of single-layer steel grid shells has been demonstrated

Surface	Number of inner nodes	Load bearing capacity [kN/m <sup>2</sup> ]			
		Initial	Relaxed	Optimal	Improvement [%]
Dome, H/L=0,2	10	1,99	2,08	2,37	14
	10 sym.	1,99	2,08	2,35	14
	42	7,96	8,21	8,25	4
Free-form #1, L=25m	7	1,92	1,88	2,46	28
	14	2,06	2,14	4,11	100
	29	3,56	-	4,23	19
Free-form #2, L=26m	24	5,45	7,06	7,16	31

Domes + freeform surfaces; coarse + dense meshes; various beam lengths

**Beam length and failure modes are different!**

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Free-form #2, L=26m	24	5,45	7,06	7,16	31

1. Improvement achieved by optimization is highly dependant on failure modes,
2. Understanding the failure modes is important – generally failure is different for initial and optimal grid pattern → local to global
3. More efficient for coarse meshes with less node number, surface is not approximated well enough
4. Dome: symmetry → aesthetic results
5. Higher node numbers for freeform structures: the mesh is distorted for practical use  
Optimality criteria should be more complex (e.g. including maximal beam length)
6. Dense meshes: in certain cases (probably depending on failure mode) relaxation results in the same mesh as the optimization – fast process

## Optimization

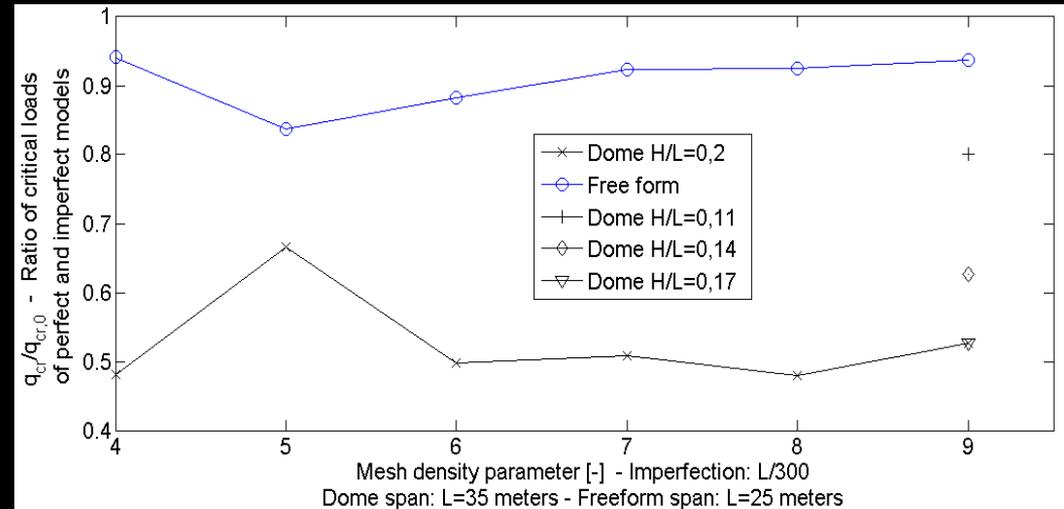
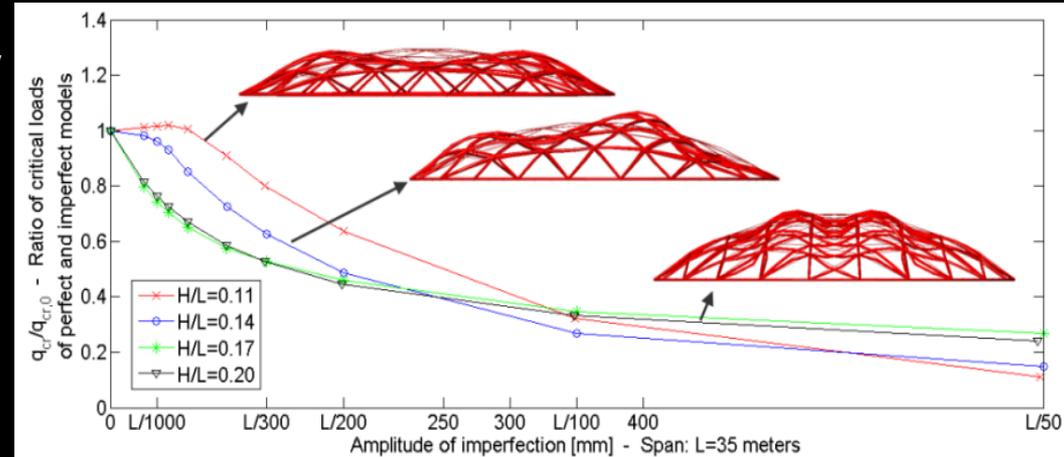
- Fitness function
- Very slow
- More realistic load cases – multidisciplinary optimization

## Settings of arc-length method

## Imperfection

- Exact nonlinear analysis
- First eigenmode
- High effect on  $q_{cr}$ : 20÷90 %
- Depends on:
  - Surface
  - Mesh density, beam length
  - Scale

Imperfection sensitivity analysis before each optimization process



- Fan, F., Cao, Z., Shen, S.: Elasto-plastic stability of single-layer reticulated shells. In: Thin-Walled Structures (2010) Vol. 48, pp. 827-836
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- Dimcic, M.: Structural Optimization of Grid Shells based on Genetic Algorithms, Ph.D. dissertation. (2011)
- Kollár, L., Hegedűs, L.: Analysis and Design of Space Frames by the Continuum Method. Akadémiai Kiadó, Budapest, Hungary (1985)
- Toğan V., Daloğlu AT.: Optimization of 3d trusses with adaptive approach in genetic algorithms. In: Engineering Structures (2006) Vol. 28, pp. 1019–1027



Divatcsarnok, Budapest, 2009

## Thank you for your attention!

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